Review of the paper
Modern space-time and undecidability
by R. Gambini and J. Pullin
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This is a short but very dense paper, highly conceptual in its contents. The authors are two senior researchers, internationally known for their outstanding work in relativity theory and quantum gravity. Fortunately one does not have to be an expert in those fields in order to understand their paper. One need only know elementary quantum mechanics and general relativity; the exposition is written in a very clear style. The main point addressed is the following question: *How do quantum notions, as applied to spacetime, alter our views of quantum mechanics?* This point of view is complementary (in Bohr's sense of the word) to the widespread opinion that, given classical general relativity on the one hand, and quantum mechanics on the other, what remains to be done is to *quantise* gravity—an enterprise (the quantisation of gravity) that has kept theoretical physicists busy for the last 80 years, with varying degrees of success. In loose terms, this paper could be seen as a step towards *relativising* the quantum.

In its usual formulation, quantum mechanics relies on the idealisation that all measuring devices are perfectly classical apparatuses, not subject to quantum fluctuations. This is however not true, as everything within the Universe, measuring devices included, is subject to some level of quantum fluctuations. When measuring spacetime, this statement implies that neither clocks nor rulers can be perfectly classical. Rather, they are subject to limitations on their accuracy; one cannot measure space and time beyond a minimum level of uncertainty. For the rest of the paper the authors concentrate their attention on clocks as measuring devices that are themselves also subject to the laws of quantum mechanics.[1] They call these apparatuses *real* clocks, as opposed to *classical* clocks. Real time, denoted T, is the physical variable measured by a real clock. These considerations have several major implications, some practical, some conceptual.

One practical implication is that the evolution equations of quantum mechanics, when written in terms of real time T, pick up additional terms with respect to the corresponding equations when written in terms of the classical time variable t. These additional terms spoil unitarity and lead to decoherence effects. Specifically, let a certain Hamiltonian H be given to generate translations along t, and let $U(t)$ be the corresponding unitary evolution operator. Call $P_t(T)$ the probability that the resulting measurement of the clock variable T correspond to the value t. Given the density matrix ρ for a system under consideration in Schroedinger's picture, in Heisenberg's picture we have a density matrix $\rho(T)$

$$\rho(T) = \int_{-\infty}^{\infty} \mathrm{d}t\, U(t)\rho U(t)^{\dagger} P_t(T).$$

[1]Rulers as measuring devices and the corresponding uncertainties in the determination of space variables have been analysed in a previous paper by the same authors; see the references.

Unitarity is lost because $\rho(T)$ is a superposition of density matrices associated with different t's, each one of which evolves unitarily. Further assume that the real clock is semiclassical, so $P_t(T)$ can be set equal to $f(T - T_{\max}(t))$, with f a function decaying very rapidly for values of t away from the maximum of the probability distribution function $T_{\max}$. Then, to leading order one finds

$$\frac{\partial \rho(T)}{\partial T} = \mathrm{i}[\rho(T), H] + \sigma(T)[H, [H, \rho(T)]],$$

where $\sigma(T)$ is the rate of change of the width of the distribution $f(t - T_{\max}(t))$. Integrating the above and using an estimate for the function $\sigma(T)$ (*i.e.*, for the best possible accuracy of a real clock) one finds the evolution of the density matrix in the energy eigenbasis:

$$\rho(T)_{nm} = \rho(0)_{nm} \exp\left(-\mathrm{i}\omega_{nm} T\right) \exp\left(-\omega_{nm}^2 T_{\text{Planck}}^{4/3} T^{2/3}\right),$$

where $T_{\text{Planck}} = 10^{-44}$ seconds is Planck's time. A pure state will inevitably become a mixed state due to the real exponential on the right–hand side; ultimately this stems from the impossibility of having a perfectly classical clock.

At first one would expect this analysis to be relevant only in regimes in which the assumption concerning our clock (its perfect classicality) is manifestly wrong. Such regimes include the Big Bang and black hole singularities. However, the unavoidability of quantum decoherence implies that the difference between the idealised version of quantum mechanics (based on classical clocks) and the real version (based on real clocks) has deeper consequences than one might initially expect. The authors briefly discuss possible experimental scenarios in which quantum mechanics with real clocks could eventually be tested. Bose–Einstein condensates, which can contain some 10^6 atoms in coherent states, appear to be optimal testing grounds.

Moreover, there are major conceptual implications of the unavoidability of quantum decoherence, *i.e.*, of the fact that pure states evolve naturally into mixed states. The authors quote three of them: the black hole information paradox, quantum computation, and the measurement problem. They concentrate on the latter. In quantum mechanics, the fact that the system being measured abruptly falls into an eigenstate right after a measurement has been performed is referred to as *the reduction process*—which leads to *the measurement problem*: how can one explain this abrupt change of state? This is accounted for by saying that there exists an interaction between the system being measured and the environment. This interaction selects a preferred basis, *i.e.*, a particular set of quasiclassical states that commute with the Hamiltonian governing that interaction. Decoherence quickly damps superpositions between the preferred states when only the system is considered (*i.e.*, when the environment is neglected), and only classical, well–defined properties appear left to an observer.

The above explanation of the measurement problem encounters some obstacles whose exposition would take us too far afield. Suffice it to say that the fact that pure states evolve naturally into mixed states contributes to surmounting these obstacles. The usual Hamiltonians describing the interaction between the system and the environment lead to offdiagonal terms, in the density matrix, that are oscillatory functions of time. Now typical environments contain a much larger number of degrees of freedom than the systems being measured. Therefore the common period of oscillation for the offdiagonal terms to recover nonvanishing values is very large, often larger than the age of the Universe. To all intents and purposes, the offdiagonal elements of the density matrix can be taken to vanish. If this were not enough, the exponential damping due to the use of real clocks has to be thrown in as well. Thus offdiagonal terms will never see their initial values restored, no matter how long one waits. The authors turn the argument around and claim that *quantum mechanics with real clocks can, thanks to fundamental decoherence, do away with the reduction process* of standard quantum mechanics. This complementary viewpoint is ultimately sublimed in the statement of *undecidability: one cannot decide whether the physical world does in fact contain the reduction process or not*—there's even worse to come: *sometimes there might be reduction, sometimes not, as the case may be.* There is no way to tell between quantum mechanics with classical clocks *and* the reduction process, on the one hand, and quantum mechanics with real clocks and fundamental decoherence *instead of* the reduction process, on the other.

To summarise, the authors convincingly argue that quantum mechanics with real clocks, as opposed to quantum mechanics with perfectly classical clocks, introduces additional uncertainties in the form of fundamental lower limits to the accuracy of time measurements. This implies that pure states necessarily evolve into mixed states. Although the discussion presented in this paper is restricted to time, an analogous analysis can be carried out for space variables. There will be additional (spatial) decoherence due to the fact that it is impossible to have clocks perfectly synchronised across space. By the same token, there will be fundamental uncertainties in the determination of space positions. Fundamental decoherence also has far–reaching conceptual implications, since quantum mechanics with real clocks can bypass the introduction of the reduction process (the measurement problem) that causes so many headaches to practitioners of quantum mechanics with classical clocks. Admittedly, the choice between the two may be a matter of taste, and the discussion here leaves the domains of physics to enter the realm of philosophy.

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